General Lepton Mixing in Holographic Composite Higgs Models

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Abstract

We introduce a scenario of lepton mixing in holographic composite Higgs models based on non-abelian discrete symmetries of the form $G_f = X \times \mathbf{Z}_N$, broken to $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N$ in the elementary sector and to $\mathbf{Z}_N^{(D)}$ in the composite sector with $\mathbf{Z}_N^{(D)}$ being the diagonal subgroup of a $\mathbf{Z}_N \subset X$ and the external $\mathbf{Z}_N$. By choosing $X = \Delta(96)$ or $\Delta(384)$, a non-vanishing θ_{13} of order 0.1 is naturally obtained. We apply our considerations to a 5D model in warped space for the particular cases of $X = S_4, A_5, \Delta(96)$ and $\Delta(384)$ and $N = 3$ or 5. Lepton flavour violating processes and electric dipole moments are well below the current bounds, with the exception of $\mu \to e\gamma$ that puts a very mild constraint on the parameter space of the model, for all presented choices of G_f.

1 Introduction

The data accumulated in neutrino experiments over the past years clearly show that lepton and quark mixing are vastly different. Several successful explanations for a lepton mixing pattern with two large angles and a small one in terms of a flavour symmetry can be found in the literature. The most prominent pattern is tri-bimaximal (TB) mixing [1] which can be elegantly derived with the help of the symmetries A_4 [2] and S_4 [3]. Recently, the T2K [4] and MINOS [5] Collaborations published indication that the lepton mixing angle θ_{13} is non-zero. According to global fits [6, 7, 8], the best fit value of θ_{13} is around $0.1 \div 0.2$ and its value is different from zero at the 3σ level [6, 7]. In the light of this, many of the models predicting TB mixing become disfavoured, because the deviation from $\theta_{13} = 0$ necessary to accommodate the best fit value of $\theta_{13} \sim 0.1 \div 0.2$ is too large to be explained by sub-leading corrections. Therefore, new lepton mixing patterns with non-vanishing θ_{13} based on discrete non-abelian symmetries have recently been put forward [9]. The key assumptions [9] (see also [3]) are that the neutrino and the charged lepton mass matrices are invariant under two distinct subgroups G_ν and G_e of a flavour group G_f, respectively, and that left-handed (LH) leptons are in an irreducible triplet representation of G_f. Non-trivial lepton mixing is determined through the relative embedding of G_ν and G_e into G_f. In contrast, lepton masses remain unconstrained.

The symmetry breaking pattern of G_f as proposed in [3, 9] is naturally realized in Composite Higgs Models (CHM), where the group G_f is broken to G_ν in the elementary and to G_e in the composite sector.[1] In fact, a concrete realization of this scenario was already introduced in CHM in [11], for the particular case $G_f = S_4 \times \mathbf{Z}_3$, leading to TB mixing and thus $\theta_{13} = 0$.

Aim of this paper is to generalize the scenario [11] in order to include lepton mixing patterns that, like in [9], lead to non-vanishing θ_{13} of order $0.1 \div 0.2$. More precisely, we consider a set-up in which the discrete flavour group is $G_f = X \times \mathbf{Z}_N$, with X being a non-abelian group. The additional cyclic symmetry $\mathbf{Z}_N$ is in general needed to keep the natural explanation of the fermion mass hierarchy given by the Holographic CHM (HCHM) or their five-dimensional (5D) realizations. The pattern of flavour symmetry breaking is driven by symmetry considerations only, and no specific sources of flavour breaking are introduced. We focus on a scenario in which the Standard Model (SM) neutrinos are Majorana fermions and the type I see-saw mechanism explains the smallness of their masses. However, a similar analysis also applies to a scenario, introduced in [11], in which neutrinos are Dirac fermions.

We choose as remnant symmetry G_ν in the elementary sector $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N$, while G_e of the composite sector is taken to be $\mathbf{Z}_N^{(D)}$, the diagonal subgroup of a $\mathbf{Z}_N \subset X$ and the external $\mathbf{Z}_N$. As discussed already in [11], a large breaking of G_f in the composite sector is favoured for charged leptons, because in this way large deviations from the SM $Z\tau\bar{\tau}$ coupling are suppressed. At the same time, the breaking of G_f in the composite sector affecting neutrinos is required to be small, in order to not perturb too much the lepton mixing pattern determined by the choice of G_f, G_e and G_ν.

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[1]Models in warped space making use of the discrete symmetry A_4 can be found in [10].

After a general presentation of our set-up in terms of 4D HCHM, we study the 5D models introduced in [11] in more detail for the choices $(X, \mathbf{Z}_N) = (S_4, \mathbf{Z}_3)$, $(A_5, \mathbf{Z}_5)$, $(\Delta(96), \mathbf{Z}_3)$ and $(\Delta(384), \mathbf{Z}_3)$. The full discrete symmetry of the 5D models is actually $G_f \times Y$, where $Y = \mathbf{Z}_3' \times \mathbf{Z}_3''$ is a flavour-independent factor useful to minimize the number of allowed terms. Keeping the prediction of the solar mixing angle θ_{12} within the experimentally allowed 3σ range requires that the flavour symmetry breaking at the IR brane for neutrinos should be smaller than 10% in all models, unless a $\mathbf{Z}_2$ exchange symmetry is imposed on the IR brane, in which case no constraint occurs. The corrections to the mixing angles θ_{23} and θ_{13} are generically smaller. In the case of $X = \Delta(96)$ they help to improve the accordance of the predicted and the measured value of θ_{23}. In the cases of $X = S_4$ and $X = A_5$, in which the unperturbed value of θ_{13} vanishes, such corrections are not enough to generate a value of θ_{13} of order $0.1 \div 0.2$, as favoured by the latest experimental data and global fit analyses. Overall, the patterns derived with $X = \Delta(384)$ describe the data in the best way. Although the neutrino mass spectrum is not predicted, a normally ordered spectrum is preferred in the 5D models, because corrections to the solar mixing angle are under much better control in this case.

For all choices of G_f which we discuss, most of the Lepton Flavour Violating (LFV) processes for charged leptons are below the current experimental bounds for masses of the first Kaluza-Klein (KK) gauge resonances of order 3.5 TeV, roughly the lowest scale allowed by electroweak considerations. The main source of such processes are Boundary Kinetic Terms (BKT) for fermions at the UV brane. The most important bound comes from the radiative decay $\mu \to e\gamma$, $BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$ [12], and is passed in most of the parameter space, while the expected future bound constrains our model. We also argue that Electric Dipole Moments (EDMs) for charged leptons are negligibly small.

The structure of the paper is as follows. In section 2 we generalize the set-up of [11] to include generic lepton mixing patterns arising from a non-trivial breaking of a flavour symmetry G_f. In section 3 we apply our considerations to the 5D Majorana model of [11] and discuss the results for lepton mixing as well as constraints coming from charged LFV decays and lepton EDMs. We conclude in section 4. The relevant group theory of S_4, A_5, $\Delta(96)$ and $\Delta(384)$ and an explicit choice of basis for their generators can be found in the appendix.

2   General Set-up

The set-up we consider is closely related to the one introduced in [11] for the particular choice of the discrete group $S_4 \times \mathbf{Z}_3$. We mainly emphasize here the key differences with respect to [11], referring the reader to [11] for further details. We consider in this paper only CHM with Majorana neutrinos, since they overall seem to perform better than the CHM with Dirac neutrinos, but similar considerations apply to the latter case as well.

The Lagrangian of the CHM consists of an elementary, a composite and a mixing sector [13]:

$$\mathcal{L}_{tot} = \mathcal{L}_{el} + \mathcal{L}_{comp} + \mathcal{L}_{mix} \,. \tag{2.1}$$

We assume that $\mathcal{L}_{mix}$ is invariant under the discrete flavour symmetry $G_f = X \times \mathbf{Z}_N$, with X a non-abelian group[2] which has the following features, see [3, 9]: i) it contains, at least, one (faithful) irreducible three-dimensional representation $\mathbf{3}$, as which LH leptons and right-handed (RH) neutrinos transform,[3] and ii) it contains $\mathbf{Z}_2 \times \mathbf{Z}_2$ and $\mathbf{Z}_N$ as non-commuting subgroups. The former requirement ensures the possibility to determine all mixing angles through the choice of G_f, G_e and G_ν, while the latter ensures that the resulting mixing pattern is non-trivial, because lepton mixing corresponds to the mismatch in the embedding of the two subgroups $\mathbf{Z}_2 \times \mathbf{Z}_2$ and $\mathbf{Z}_N$ into X.

The symmetry G_f is broken in the elementary sector to $G_\nu = \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N$, where $\mathbf{Z}_2 \times \mathbf{Z}_2 \subset X$, and in the composite sector to $G_e = \mathbf{Z}_N^{(D)}$, the diagonal subgroup of the external $\mathbf{Z}_N$ and $\mathbf{Z}_N \subset X$. Thus, all terms of $\mathcal{L}_{el}$ are invariant under G_ν, while all terms of $\mathcal{L}_{comp}$ under G_e.

G_e is chosen as $\mathbf{Z}_N^{(D)}$ with $N \geq 3$ in order to distinguish the three generations of charged leptons and consequently to explain the observed hierarchy among their masses. More precisely, it has to be the diagonal subgroup of $\mathbf{Z}_N \subset X$ and the external $\mathbf{Z}_N$ because LH leptons have to be assigned to a $\mathbf{3}$ of X for lepton mixing, while RH charged leptons transform trivially under X and carry only non-trivial charge under the external $\mathbf{Z}_N$. Obviously, in order to distinguish among the three generations, their charges have to be different. We consider the group $\mathbf{Z}_N$ for simplicity. In a more general set-up this group can be replaced by a product of cyclic symmetries, such as $\mathbf{Z}_2 \times \mathbf{Z}_2$. If the group X has three or more inequivalent one-dimensional representations, the additional cyclic group factor might be abandoned, because it is then possible to distinguish the three generations of (RH) charged leptons with the help of X alone.

G_ν consists of a Klein group $\mathbf{Z}_2 \times \mathbf{Z}_2$ and of the external $\mathbf{Z}_N$. The Klein group is the maximal symmetry preserved by a Majorana mass matrix in the case of three neutrinos [3], and at the same time it can guarantee the existence of three independent parameters corresponding to the neutrino masses.[4] The external $\mathbf{Z}_N$ does not play any direct role for the generation of lepton mixing, because neither LH leptons nor RH neutrinos transform under it, but it automatically keeps the kinetic terms of RH charged leptons flavour diagonal in the elementary sector (see below).[5]

In the basis in which the generator G_N of $\mathbf{Z}_N \subset X$ is diagonal for $\mathbf{3}$,

$$G_N = \begin{pmatrix} \omega_N^{n_e} & 0 & 0 \\ 0 & \omega_N^{n_\mu} & 0 \\ 0 & 0 & \omega_N^{n_\tau} \end{pmatrix}, \quad \text{with} \quad n_e \neq n_\mu \neq n_\tau, \quad \omega_N = e^{2\pi i/N}, \tag{2.2}$$

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[2]The group X can in principle be infinite, but all examples we present in the following make use of a finite X.

[3]We assume for simplicity that LH leptons and RH neutrinos transform in the same way under G_f.

[4]In the case of Dirac neutrinos G_ν is not constrained to contain a Klein group, since the symmetry preserved by a Dirac neutrino mass matrix can be any product of cyclic symmetries which allows to distinguish among the three generations. The simplest case is then that G_ν is a product of a cyclic symmetry $\mathbf{Z}_M$ and the external $\mathbf{Z}_N$ with $M, N \geq 3$.

[5]In a set-up without the external $\mathbf{Z}_N$ the RH charged leptons should transform as three distinct singlets under the subgroup $\mathbf{Z}_2 \times \mathbf{Z}_2$ contained in G_ν (and thus under X) in order to keep their kinetic terms flavour diagonal in the elementary sector.

the generators G_1 and G_2 of $\mathbf{Z}_2 \times \mathbf{Z}_2$ are of the form

$$G_1 = V G_1^{diag} V^\dagger, \qquad G_2 = V G_2^{diag} V^\dagger, \tag{2.3}$$

with

$$G_1^{diag} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad G_2^{diag} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{2.4}$$

and V a unitary matrix.

The elementary sector contains three generations of SM LH and RH leptons l_L^α, l_R^α and three RH neutrinos ν_R^α. The LH leptons l_L^α and the RH neutrinos ν_R^α transform as $(\mathbf{3}, 1)$ under $X \times \mathbf{Z}_N$, while the RH charged leptons l_R^α transform as $(\mathbf{1}, \omega_N^{n_\alpha})$. The elementary Lagrangian is

$$\mathcal{L}_{el} = \bar{l}_L^\alpha i \slashed{D} l_L^\alpha + \bar{l}_R^\alpha i \slashed{D} l_R^\alpha + \bar{\nu}_R^\alpha i \slashed{\partial} \nu_R^\alpha - \frac{1}{2} (\overline{\nu_R^c}^\alpha M_{\alpha\beta} \nu_R^\beta + h.c.), \tag{2.5}$$

where M is the most general mass matrix invariant under $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N$, of the form

$$M = V^\star M_D V^\dagger, \tag{2.6}$$

with V as in (2.3) and M_D a diagonal matrix containing three independent complex parameters.

The composite sector is an unspecified strongly coupled theory, that gives rise, among other states, to a composite SM Higgs field and vector-like fermion resonances Ψ mixing with the SM fields. The fermion mixing Lagrangian $\mathcal{L}_{mix}$ is

$$\mathcal{L}_{mix} = \frac{\lambda_{l_L}}{\Lambda^{\gamma_{lL}}} \bar{l}_L^\alpha \Psi_{l_L,R}^\alpha + \frac{\lambda_{l_R}^\alpha}{\Lambda^{\gamma_{lR}^\alpha}} \bar{l}_R^\alpha \Psi_{l_R,L}^\alpha + \frac{\lambda_{\nu_R}}{\Lambda^{\gamma_{\nu R}}} \bar{\nu}_R^\alpha \Psi_{\nu_R,L}^\alpha + h.c. \tag{2.7}$$

where Λ is the UV cut-off scale of the composite sector and $\Psi_{l_L}^\alpha$, $\Psi_{l_R}^\alpha$ and $\Psi_{\nu_R}^\alpha$ are fermion resonances transforming under G_f in the same way as l_L^α, l_R^α and ν_R^α, respectively. The mixing parameters λ_{l_L} and λ_{ν_R} are flavour universal, because two triplets of X are coupled to each other, while $\lambda_{l_R}^\alpha$ are flavour diagonal, but non-universal, since RH charged leptons and $\Psi_{l_R}^\alpha$ are singlets under X. Integrating out the composite sector gives rise to the following charged lepton mass matrix (in left-right convention, $\bar{\psi}_L M_l \psi_R$):

$$M_{l,\alpha\beta} \sim b_\alpha v_H \lambda_{l_L} \lambda_{l_R}^\alpha \delta_{\alpha\beta} \left(\frac{\mu}{\Lambda}\right)^{\gamma_{lR}^\alpha + \gamma_{lL}}, \tag{2.8}$$

where $v_H \simeq 250$ GeV, μ is the $\mathcal{O}(\text{TeV})$ scale at which the composite theory becomes strongly coupled and b_α are $\mathcal{O}(1)$ coefficients. In this basis the charged lepton mass matrix is flavour diagonal and non-trivial mixing is encoded in the light neutrino mass matrix. The latter arises upon integrating out the RH neutrinos ν_R^α

$$M_{\nu,\alpha\beta} \simeq \hat{b}_\alpha \hat{b}_\beta v_H^2 \lambda_{l_L}^2 \lambda_{\nu_R}^2 \left(\frac{\mu}{\Lambda}\right)^{2(\gamma_{\nu R}+\gamma_{lL})} M_{\alpha\beta}^{-1} \simeq \hat{b}^2 v_H^2 \lambda_{l_L}^2 \lambda_{\nu_R}^2 \left(\frac{\mu}{\Lambda}\right)^{2(\gamma_{\nu R}+\gamma_{lL})} \left(V M_D^{-1} V^T\right)_{\alpha\beta} \tag{2.9}$$

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with $\hat{b}_\alpha$ being order one coefficients. The second relation holds in the limit of universal $\hat{b}_\alpha$. Only in this limit the lepton mixing matrix U_{PMNS} is given by

$$U_{PMNS} = V \,. \tag{2.10}$$

Obviously, a sensible choice of V implies that the resulting mixing angles are in good agreement with the experimental data. Deviations from the universality of $\hat{b}_\alpha$ lead to corrections of the lepton mixing angles and, in order to keep the accordance with experimental data, $\hat{b}_\alpha$ generically have to be universal at a level of $\lesssim 10\%$ (depending slightly on the choice of V). This condition is equivalent to requiring that the breaking of G_f in the composite sector should be small for neutrinos. On the other hand, it can be of order one for charged leptons, because the values of b_α do not have a direct impact on the lepton mixing.

The remnant symmetry $\mathbf{Z}_N^{(D)}$ renders all couplings flavour diagonal in the composite sector and flavour violation is only present in (2.5). All flavour changing processes are then negligibly small, since they are suppressed by the large Majorana mass of the RH neutrinos. The main source of flavour violation arises from the elementary sector, if the most general kinetic terms of the SM fermions compatible with the flavour symmetries are taken into account. These are of the form

$$\bar{l}_L(1+Z_l)i\slashed{D}l_L + \bar{l}_R(1+\tilde{Z}_l^D)i\slashed{D}l_R + \bar{\nu}_R(1+Z_\nu)i\slashed{\partial}\nu_R \tag{2.11}$$

with $Z_l = V Z_l^D V^\dagger$, $Z_\nu = V Z_\nu^D V^\dagger$ and Z_l^D, $\tilde{Z}_l^D$ and Z_ν^D diagonal matrices. As explained above, the $\mathbf{Z}_N$ contained in G_ν forbids flavour violating kinetic terms for l_R^α. Non-trivial LFV processes (and further corrections to the lepton mixing (2.10)) are now generated and are proportional to the non-diagonal entries of the matrices Z_l and Z_ν in (2.11). In the limit in which the composite sector is G_f-invariant, one can go to a basis in which the whole Lagrangian $\mathcal{L}_{tot}$ in (2.1) is flavour diagonal, since in this limit all couplings and mass terms of fermion resonances Ψ forming triplets under G_f are flavour universal. Thus, the actual amount of LFV is controlled by the size of the flavour violation in the elementary *and* the flavour non-universality in the composite sector and is consequently suppressed with respect to an anarchic scenario with no flavour symmetries.

Without further constraints on the breaking of G_f in the elementary sector, the elements of $Z_{l,\nu}^D$ are expected to be uncorrelated $\mathcal{O}(1)$ parameters, possibly leading to too large flavour violating processes and corrections to the lepton mixing. The actual effect induced by $Z_{l,\nu}$, however, depends on the degree of compositeness of l_L^α and ν_R^α (in turn determined by the mass mixing terms λ_{l_L} and λ_{ν_R}), since the kinetic terms of the elementary fields always receive a contribution coming from the strongly coupled sector, when the fermion resonances Ψ are integrated out (c.f. (2.7) of [11]). Thus we have to rescale the fields to canonically normalize their kinetic terms, $\psi \to \psi/\sqrt{A_\psi}$. If the contribution from the composite sector dominates, $A_\psi \gg 1$, the rescaled parameters $Z_{l,\nu}/A_{l,\nu}$ become suppressed. As explained in [11], λ_{ν_R} is a relevant coupling and ν_R^α are mostly composite fields. The effect of Z_ν is thus negligible, being suppressed by the large value of A_ν. The mixing term λ_{l_L}, on the other hand, should be irrelevant, otherwise too large deviations from the SM gauge couplings of l_L^α would occur (see (2.14) below), but it can be very close to be marginal, $\gamma_{lL} \simeq 0$. In this case the kinetic term of

l_L^α can still be dominated by the contribution coming from the composite sector due to a large logarithmic running from the scale Λ to μ, and again the rescaled parameters Z_l/A_l turn out to be small (compare (3.17)). Summarizing, the flavour violating effects and the corrections to lepton mixing induced by $Z_{l,\nu}$ are naturally suppressed by the dynamics in the composite sector.

Flavour symmetries can be important also for flavour conserving observables, such as the EDMs. Using standard conventions, we denote the charged lepton EDMs d_α as the coefficient of the dimension five operator $(-i/2)\bar{l}_\alpha \sigma^{\mu\nu}\gamma_5 l_\alpha F_{\mu\nu}$. Being the latter a CP-odd operator, EDMs can be generated, if CP violation is present in the lepton sector. In absence of flavour symmetries, a rough one-loop estimate, assuming generic masses and complex couplings in the composite sector, gives

$$d_\alpha \sim \frac{eM_{l,\alpha}}{16\pi^2}\frac{Y^2}{m_\Psi^2}\,, \tag{2.12}$$

where $M_{l,\alpha}$ are the charged lepton masses, Y represents a typical $\mathcal{O}(1)$ Yukawa coupling involving the composite fermion resonances and m_Ψ their typical mass. The appearance of $M_{l,\alpha}$ in (2.12) is a consequence of the partial compositeness of the SM fermions which implies that the EDMs vanish unless both mass mixing for LH and RH charged leptons are inserted in the one-loop diagram. Despite the explicit dependence on the charged lepton mass, the strongest bound comes from the EDM of the electron, $|d_e| \lesssim 10^{-27}e$ cm [14], giving $m_\Psi \gtrsim 10\,Y$ TeV. In our scenario, however, the leading order result (2.12) vanishes for canonical kinetic terms (2.5), because the relevant non-trivial phases can be removed through field redefinitions.[6] For general kinetic terms (2.11) and flavour symmetry breaking in the composite sector, the leading order term no longer vanishes and is estimated to be, assuming again arbitrary complex couplings in the composite sector,

$$d_\alpha \sim \frac{eM_{l,\alpha}}{16\pi^2}\frac{Y^2}{m_\Psi^2}\frac{\delta m_\Psi}{m_\Psi}Z_l^2\,, \tag{2.13}$$

where δm_Ψ is the inter-generational mass splitting of the fermion resonances. The last two terms in (2.13) are essential suppression factors that can significantly reduce the size of d_α.

Let us conclude this section by showing how deviations from the SM $Zl_\alpha\bar{l}_\alpha$ couplings disfavour a small breaking of the flavour symmetry in the composite sector for charged leptons, $b_\alpha \ll 1$ in (2.8). For simplicity we set $Z_l = \tilde{Z}_l^D = Z_\nu = 0$, their effect being sub-leading. The coupling deviations arise from mixing of the SM leptons with fermion resonances, induced by the mixing terms in (2.7). One schematically has

$$\frac{\delta g_{l_{\alpha i}}}{g_{l_{\alpha i}}} \sim \frac{v_H^2}{m_\Psi^2}(\lambda_{l_i}^\alpha)^2\left(\frac{\mu}{\Lambda}\right)^{2\gamma_{li}^\alpha},\quad i = L, R\,. \tag{2.14}$$

Using the charged lepton mass formula (2.8), we can write

$$\frac{\delta g_{l_{\alpha L}}}{g_{l_{\alpha L}}}\frac{\delta g_{l_{\alpha R}}}{g_{l_{\alpha R}}} \sim \frac{M_{l,\alpha}^2}{m_\Psi^2}\frac{v_H^2}{m_\Psi^2}\frac{1}{b_\alpha^2}\,. \tag{2.15}$$

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[6]This is not true in general, if one considers non-minimal scenarios, in which for example SM fermions mix with more than one state of the composite sector.

The tension between having parametrically small b_α and sufficiently small gauge coupling deviations is obvious from (2.15). This is particularly important for the tau lepton due to its larger mass. Deviations from the SM gauge couplings for (charged and neutral) leptons have been constrained by LEP at the per mille level [15]. For fermion resonances at the TeV scale, we see that the right-hand side of (2.15) is below 10^{-6} for b_τ slightly below one. Notice that one might actually cancel the leading term (2.14) in $\delta g_{l_{\alpha L}}$ for either charged leptons or neutrinos using appropriate symmetries [16], but not both at the same time. The relation (2.15) is then always valid for at least one of the two components of the LH doublet l_L^α.

3 5D Realizations

Models with gauge-Higgs unification in warped space based on an $SO(5) \times U(1)_X$ gauge symmetry [13, 17] are an explicit and particularly interesting weakly coupled description of the scenario outlined in section 2.[7] We consider flavour groups of the form:

$$G_f \times Y \, . \tag{3.1}$$

The additional discrete symmetry Y,

$$Y = \mathbf{Z}_3' \times \mathbf{Z}_3'' \, , \tag{3.2}$$

is introduced in order to minimize the number of couplings in the bulk and at the branes. We present four examples in the following leading to different results for the lepton mixing angles. Our first choice of G_f is

$$G_f = S_4 \times \mathbf{Z}_3 \tag{3.3}$$

which is discussed in [11] and leads to TB mixing, see (3.10). We repeat its analysis here for completeness. The choice

$$G_f = A_5 \times \mathbf{Z}_5 \tag{3.4}$$

gives rise to the so-called Golden Ratio (GR) mixing in which the solar mixing angle θ_{12} is determined in terms of $\phi = \frac{1}{2}(1 + \sqrt{5})$ [19], while vanishing θ_{13} and maximal θ_{23} are predicted, see (3.11). We include its discussion, albeit it might seem to be disfavoured due to the prediction $\theta_{13} = 0$, in order to show an example in which the external $\mathbf{Z}_N$ factor is different from $\mathbf{Z}_3$. As third and forth choice, we discuss the cases

$$G_f = \Delta(96) \times \mathbf{Z}_3 \, , \quad G_f = \Delta(384) \times \mathbf{Z}_3 \, , \tag{3.5}$$

since it has recently been shown that these groups can naturally lead to $\theta_{13} \sim 0.1 \div 0.2$ [9]. They give rise to two inequivalent mixing patterns each (differing in the value of the angle θ_{23}), and thus we get in total four different possibilities. Following the notation of [9], we call them M1 and M2 for $\Delta(96)$, see (3.12) and (3.13), and M3 and M4 for $\Delta(384)$, see (3.14) and (3.15), respectively.

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[7]Notice that warping is not a necessary ingredient. For viable models in flat space see [18].

	Bulk	UV	IR
	$G_f \times Y$	$G_{f,\mathrm{UV}} \times Y_{\mathrm{UV}}$	$G_{f,\mathrm{IR}} \times Y_{\mathrm{IR}}$
$\xi_{l,\alpha}$	$(\mathbf{3}, 1, \omega_3, \omega_3)$	$(1, -1, 1, \omega_3)$ $(-1, 1, 1, \omega_3)$ $(-1, -1, 1, \omega_3)$	$(\omega_N^{n_\alpha}, \omega_3)$
$\xi_{e,\alpha}$	$(\mathbf{1}, \omega_N^{n_\alpha}, \omega_3, \omega_3)$	$(1, 1, \omega_N^{n_\alpha}, \omega_3)$	$(\omega_N^{n_\alpha}, \omega_3)$
$\xi_{\nu,\alpha}$	$(\mathbf{3}, 1, \omega_3, 1)$	$(1, -1, 1, 1)$ $(-1, 1, 1, 1)$ $(-1, -1, 1, 1)$	$(\omega_N^{n_\alpha}, \omega_3)$

Table 1: Transformation properties of the 5D multiplets $\xi_{l,\alpha}$, $\xi_{e,\alpha}$ and $\xi_{\nu,\alpha}$ under $G_f \times Y$ and the subgroups $G_{f,\mathrm{UV}} \times Y_{\mathrm{UV}}$ and $G_{f,\mathrm{IR}} \times Y_{\mathrm{IR}}$. The values of N and n_α for each group can be found in table 2.

The flavour symmetry is broken at the UV and IR branes to

$$G_{f,\mathrm{UV}} \times Y_{\mathrm{UV}} = \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N \times \mathbf{Z}_3'' , \quad G_{f,\mathrm{IR}} \times Y_{\mathrm{IR}} = \mathbf{Z}_N^{(D)} \times \mathbf{Z}_3' , \tag{3.6}$$

with $N = 3$ in the case of $X = S_4$, $\Delta(96)$ and $\Delta(384)$, and $N = 5$ for A_5.

The lepton particle content of the model is identical to the one of [11] with respect to the gauge group: three 5D bulk fermions $\xi_{l,\alpha}$, $\xi_{e,\alpha}$ and $\xi_{\nu,\alpha}$, in the fundamental, adjoint and singlet representations of $SO(5)$ are introduced (for details of the notation see [11]). All of them have vanishing $U(1)_X$ charge. Their flavour properties are reported in tables 1 and 2.

The most general $G_{f,\mathrm{IR}} \times Y_{\mathrm{IR}}$ invariant mass terms at the IR brane are

$$-\mathcal{L}_{\mathrm{IR}} = \left(\frac{R}{R'}\right)^4 \sum_{\alpha=e,\mu,\tau} \left(m_{\mathrm{IR},\alpha}^l \left(\overline{\tilde{L}}_{1,\alpha L} \tilde{L}_{2,\alpha R} + \overline{L}_{\alpha L} \hat{L}_{\alpha R}\right) + m_{\mathrm{IR},\alpha}^\nu \, \overline{\hat{\nu}}_{\alpha L} \nu_{\alpha R} + h.c. \right) . \tag{3.7}$$

In the particular bases chosen for the different groups G_f, see section 2 and the appendix, these terms are flavour diagonal. The fields $\tilde{L}_{1,\alpha}$, L_α and $\hat{\nu}_\alpha$ are components of the 5D multiplet $\xi_{l,\alpha}$, $\tilde{L}_{2,\alpha}$ and $\hat{L}_\alpha$ are contained in $\xi_{e,\alpha}$ and $\nu_\alpha = \xi_{\nu,\alpha}$. The only $G_{f,\mathrm{UV}} \times Y_{\mathrm{UV}}$ invariant mass terms at the UV brane are Majorana mass terms for RH neutrinos:

$$-\mathcal{L}_{\mathrm{UV}} = \frac{1}{2} \overline{\nu_{\alpha R}^c} \mathcal{M}_{\mathrm{UV},\alpha\beta} \nu_{\beta R} + h.c. \tag{3.8}$$

with

$$\mathcal{M}_{\mathrm{UV}} = V^\star m_{\mathrm{UV}} V^\dagger , \tag{3.9}$$

$m_{\mathrm{UV}} = \mathrm{diag}\,(m_{\mathrm{UV},e}\,, m_{\mathrm{UV},\mu}\,, m_{\mathrm{UV},\tau})$ and V as in (2.3). Up to (removable) phases and signs,

X	S_4	A_5	$\Delta(96)$, M1	$\Delta(96)$, M2	$\Delta(384)$, M3	$\Delta(384)$, M4
N	3	5	3	3	3	3
n_α	(0,2,1)	(0,1,4)	(2,1,0)	(2,0,1)	(1,2,0)	(1,0,2)

Table 2: Values of N and n_α for the different choices of non-abelian discrete groups X and mixing patterns.

the explicit form of V is as follows:

$$
X = S_4 : V = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}, \tag{3.10}
$$

$$
X = A_5 : V = U_{GR} = \begin{pmatrix} \cos\theta_{12}^{GR} & -\sin\theta_{12}^{GR} & 0 \\ \frac{\sin\theta_{12}^{GR}}{\sqrt{2}} & \frac{\cos\theta_{12}^{GR}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}^{GR}}{\sqrt{2}} & \frac{\cos\theta_{12}^{GR}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad \text{with} \quad \tan\theta_{12}^{GR} = 1/\phi, \tag{3.11}
$$

$$
X = \Delta(96), \text{M1} : V = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2}(\sqrt{3}+1) & 1 & \frac{1}{2}(\sqrt{3}-1) \\ \frac{1}{2}(\sqrt{3}-1) & 1 & -\frac{1}{2}(\sqrt{3}+1) \\ 1 & 1 & 1 \end{pmatrix}, \tag{3.12}
$$

$$X = \Delta(96), \text{M2} : V \text{ equal to the one of M1 with } 2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ rows exchanged}, \tag{3.13}$$

$$
X = \Delta(384), \text{M3} : V = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{1}{2}\sqrt{4+\sqrt{2}+\sqrt{6}} & 1 & -\frac{1}{2}\sqrt{4-\sqrt{2}-\sqrt{6}} \\ \frac{1}{2}\sqrt{4+\sqrt{2}-\sqrt{6}} & 1 & -\frac{1}{2}\sqrt{4-\sqrt{2}+\sqrt{6}} \\ \sqrt{1-\frac{1}{\sqrt{2}}} & 1 & \sqrt{1+\frac{1}{\sqrt{2}}} \end{pmatrix}, \tag{3.14}
$$

$$X = \Delta(384), \text{M4} : V \text{ equal to the one of M3 with } 2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ rows exchanged}. \tag{3.15}$$

In order to discuss the result for lepton mixing analytically, we consider charged lepton and neutrino mass matrices in the Zero Mode Approximation (ZMA), including the effect of the dominant flavour violating BKT

$$
\mathcal{L}_{BKT} = \bar{L}_L(x,R)(R\hat{Z}_l)i\slashed{D}L_L(x,R), \tag{3.16}
$$

with $\hat{Z}_l$ being constrained by $\mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_N$ to be of the form $\hat{Z}_l = V\text{diag}(\hat{z}_{el}, \hat{z}_{\mu l}, \hat{z}_{\tau l})V^\dagger$.[8] The effective BKT relevant for the single KK modes are obtained by multiplying $\hat{Z}_l$ with the square

---

[8]The coefficients $\hat{z}_{el}$, $\hat{z}_{\mu l}$ and $\hat{z}_{\tau l}$ were denoted by z_{el}, $z_{\mu l}$ and $z_{\tau l}$ in [11]. We use the latter notation for the entries of Z_l defined in (3.17).

of their wave function profile evaluated at the UV brane. For zero modes we get, if the bulk mass parameter c_l of the fermions $\xi_{l,\alpha}$ fulfills $c_l = 1/2 + \delta_c$, at linear order in δ_c,

$$Z_l \simeq \left(\log^{-1} \frac{R'}{R} + \delta_c \right) \hat{Z}_l \simeq \left(\frac{1}{35} + \delta_c \right) \hat{Z}_l \,, \tag{3.17}$$

taking R of the order of the inverse of the reduced Planck mass and R' of the order of the inverse of the TeV scale. The matrix $\hat{Z}_l$ in (3.17) should roughly be identified with Z_l introduced in (2.11), and $1/35$ is the 5D counterpart of the suppression factor coming from the composite sector, discussed in section 2. The latter plays a crucial role in naturally suppressing most of the LFV processes well below their current experimental bounds and in keeping the corrections to the lepton mixing small. The charged lepton mass matrix in the ZMA reads, after canonical normalization of the kinetic terms (3.16) and additionally rotating LH charged leptons with $V^\dagger$ (again all relevant notation can be found in [11])

$$M_{l,\alpha\beta} = \frac{h}{\sqrt{2}R'} \, f_{c_l} f_{-c_\beta} \left(V \frac{1}{\sqrt{1 + Z_l^D}} V^\dagger \frac{m_{\mathrm{IR}}^l}{\sqrt{\rho}} \right)_{\alpha\beta} . \tag{3.18}$$

The light neutrino mass matrix, after integrating out the heavy RH neutrinos and canonically normalizing the kinetic terms (this time without rotating LH neutrinos with $V^\dagger$), is

$$M_{\nu,\alpha\beta} = \frac{h^2}{2R'^2} f_{c_l}^2 \left(\frac{R'}{R} \right)^{2c_\nu + 1} \left(\frac{1}{\sqrt{1 + Z_l^D}} V^\dagger \frac{m_{\mathrm{IR}}^\nu}{\sqrt{\rho}} V \frac{R}{m_{\mathrm{UV}}} V^T \frac{m_{\mathrm{IR}}^\nu}{\sqrt{\rho}} V^\star \frac{1}{\sqrt{1 + Z_l^D}} \right)_{\alpha\beta} . \tag{3.19}$$

In this basis the charged current is of the form

$$\bar{l}_L W^- V \nu_L \tag{3.20}$$

which coincides with the result given in (2.10) in the limit in which the BKT are set to zero and the mass parameters $m_{\mathrm{IR},\alpha}^\nu$ as well as the factors ρ_α are taken to be universal. In the phenomenologically interesting region of the parameter space in which the bulk mass parameter c_l of $\xi_{l,\alpha}$ is close to $1/2$, the mass parameters $m_{\mathrm{IR},\alpha}^l$ can be of order one without affecting considerably the universality of the parameters ρ_α and thus the results for lepton mixing. On the other hand, we still need to assume a small breaking of G_f at the IR brane in the neutrino sector, i.e.

$$m_{\mathrm{IR},\alpha}^\nu = m_{\mathrm{IR},0}^\nu (1 + \delta_\alpha) \,, \tag{3.21}$$

with $|\delta_\alpha| \ll 1$, in order to keep their impact on the mixing angles under control. As explained in [11], the parameters $\hat{b}_\alpha$ in (2.9) should be identified with the mass parameters $m_{\mathrm{IR},\alpha}^\nu$ and thus δ_α measure the non-universality of $\hat{b}_\alpha$. As we see below, $|\delta_\alpha| \lesssim 0.1$ are required, reflecting that the parameters $\hat{b}_\alpha$ have to be nearly universal. We analyze this issue in more detail in the next subsection. Alternatively, we can require the invariance of the IR localized Lagrangian under a $\mathbf{Z}_2$ exchange symmetry, under which

$$\hat{\nu}_\alpha(x, R') \leftrightarrow \nu_\alpha(x, R') \,, \quad \tilde{L}_{1,\alpha}(x, R') \leftrightarrow \tilde{L}_{2,\alpha}(x, R') \,, \quad L_\alpha(x, R') \leftrightarrow \hat{L}_\alpha(x, R') \,, \tag{3.22}$$

so that we can take $m^\nu_{\text{IR},\alpha} = m^l_{\text{IR},\alpha} = 1$ (up to an irrelevant sign per generation). We denote this constrained model as $\mathbf{Z}_2$-invariant model.

3.1 Lepton Mixing

We first discuss the phenomenological constraints on the size of δ_α, as defined in (3.21), in the ZMA at linear order in the perturbation δ_α. In doing so we set $Z^D_l = 0$ and neglect the non-universality of the parameters ρ_α, which is small for $c_l \simeq 1/2$. Then we analyze these constraints numerically by taking into account the first KK level, still without considering the effect of BKT.

We find the following analytical results, for normally ordered neutrinos with a lightest neutrino mass $m_0 = 0.01$ eV and solar and atmospheric mass square differences $\Delta m^2_{\text{sol}} = 7.59 \times 10^{-5}$ eV^2 and $\Delta m^2_{\text{atm}} = 2.40 \times 10^{-3}$ eV^2 [20]: in the case of S_4 and A_5, deviations from $\theta_{13} = 0$ and maximal θ_{23} are proportional to the breaking of $\mu - \tau$ symmetry $(\delta_\mu - \delta_\tau)$ and are nearly the same (the values in square brackets, if given, refer to A_5):

$$
\begin{aligned}
\sin \theta_{13} &\approx 0.05 \, |\delta_\mu - \delta_\tau| \,, \\
\sin^2 \theta_{23} &\approx \frac{1}{2} + 0.82 \, [0.83] \, (\delta_\mu - \delta_\tau) \,, \\
\sin^2 \theta_{12} &\approx \frac{1}{3} \, [0.28] + 1.58 \, [1.43] \, (2\delta_e - \delta_\mu - \delta_\tau) \,.
\end{aligned}
\tag{3.23}
$$

For the mixing pattern M1 arising from $\Delta(96)$ we find

$$
\begin{aligned}
\sin^2 \theta_{13} &\approx 0.04 + 0.13\delta_e - 0.11\delta_\mu - 0.03\delta_\tau \,, \\
\sin^2 \theta_{23} &\approx 0.65 - 0.02\delta_e + 0.76\delta_\mu - 0.74\,\delta_\tau \,, \\
\sin^2 \theta_{12} &\approx 0.35 + 3.09\delta_e - 0.67\delta_\mu - 2.42\delta_\tau \,.
\end{aligned}
\tag{3.24}
$$

The results for M2 are related to these by exchanging δ_μ and δ_τ and by replacing the unperturbed value of $\sin^2 \theta_{23}$ by 0.35 together with a sign change in its corrections (remember that M1 and M2 are related by the exchange of the second and third rows of V). As can be seen, for $\delta_\mu \approx \delta_\tau$ corrections to the atmospheric mixing angle become suppressed. For the mixing pattern M3 coming from $\Delta(384)$ we get

$$
\begin{aligned}
\sin^2 \theta_{13} &\approx 0.01 + 0.04\delta_e - 0.004\delta_\mu - 0.03\delta_\tau \,, \\
\sin^2 \theta_{23} &\approx 0.42 + 0.01\delta_e + 0.80\delta_\mu - 0.81\,\delta_\tau \,, \\
\sin^2 \theta_{12} &\approx 0.34 + 3.15\delta_e - 2.02\delta_\mu - 1.13\delta_\tau \,.
\end{aligned}
\tag{3.25}
$$

The mixing angles and their corrections in δ_α associated with the patterns M3 and M4 are related in the same way as those of M1 and M2 (the unperturbed value of $\sin^2 \theta_{23}$ is 0.58 in the case of M4). Again, θ_{23} only receives small corrections for $\delta_\mu \approx \delta_\tau$. As (3.23)-(3.25) show, the corrections to the solar mixing angle, the one which is experimentally determined with best precision, are generally the largest with the coefficients of δ_α being larger than one. This also implies that the above perturbative expansion makes sense only for $|\delta_\alpha| \lesssim 0.1$. As mentioned in [11], the validity of the expansion in δ_α strongly depends on m_0 and gets worse for increasing m_0. In the case
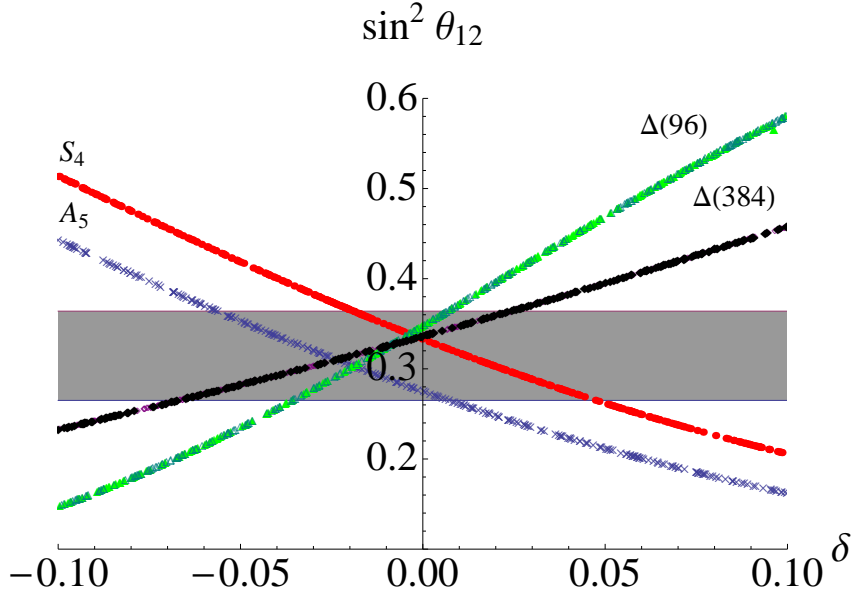
Figure 1: The solar mixing angle θ_{12} as function of the deviation δ, parametrizing the non-universality of the masses $m^{\nu}_{\mathrm{IR},\alpha}$, see (3.26). The horizontal gray band corresponds to the 3σ range as given in [6] using new reactor fluxes. The different symbols and colors distinguish the various models: S_4 (red points), A_5 (blue crosses), $\Delta(96)$ and mixing pattern M1 (light green full triangles), $\Delta(96)$ and mixing pattern M2 (dark green open triangles) , $\Delta(384)$ and mixing pattern M3 (black full diamonds) and $\Delta(384)$ and mixing pattern M4 (violet open diamonds). The curves for M1 and M2 and M3 and M4, respectively, lie on top of each other due to the parametrization chosen in (3.26). The plots refer to $c_l = 0.52$, $c_\nu = -0.365$, $h = 1/3$ and normal neutrino mass hierarchy. The masses $m_{\mathrm{UV},\alpha}$ are chosen such that the lightest neutrino mass is $m_0 = 0.01$ eV and the values of the solar and atmospheric mass square differences $\Delta m^2_{\mathrm{sol}} = 7.59 \times 10^{-5}$ eV^2 and $\Delta m^2_{\mathrm{atm}} = 2.40 \times 10^{-3}$ eV^2 [20] are reproduced using the ZMA. For simplicity, we take the IR mass terms $m^l_{\mathrm{IR},\alpha} = 1$ and set all BKT to zero. We vary $m^{\nu}_{\mathrm{IR},0}$, see (3.21), between 0.3 and 1.

of inverted mass hierarchy the coefficients multiplying the linear perturbations in θ_{12} are more than one order of magnitude bigger than the ones in (3.23)-(3.25), implying that a perturbative expansion in δ_α is not valid for any value of m_0. This behaviour is in general expected due to the near degeneracy of the two heavier neutrinos in the case of an inversely ordered mass spectrum. Note that in the limit of universal δ_α all corrections to the mixing angles vanish.

In our numerical analysis we discuss the maximal allowed size of the corrections δ_α in order to keep accordance with experimental data. We choose a particular parametrization for the deviations of $m^{\nu}_{\mathrm{IR},\alpha}$ from universality in terms of only one parameter δ:

$$S_4 \, , \, A_5 \, : \, \delta_e = 0, \delta_\mu = \delta, \delta_\tau = 0 \, ,$$
$$\Delta(96), \texttt{M1} \, , \, \Delta(384), \texttt{M3} \, : \, \delta_e = \delta, \delta_\mu = \delta, \delta_\tau = 0 \, , \qquad (3.26)$$
$$\Delta(96), \texttt{M2} \, , \, \Delta(384), \texttt{M4} \, : \, \delta_e = \delta, \delta_\mu = 0, \delta_\tau = \delta \, .$$

This particular parametrization leads to the same results for the mixing angles θ_{13} and θ_{12} in

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case of the patterns M1 and M2 as well as for M3 and M4. The atmospheric mixing angle acquires at the same time a correction which is the same in size, but opposite in sign for M1 (M3) and M2 (M4). As already obvious from the analytical results, the corrections are the largest for the angle θ_{12} whose dependence on δ we report in figure 1, together with its experimentally allowed 3σ range using the new estimate of reactor anti-neutrino fluxes [6]. Clearly, $|\delta|$ has to be smaller than 0.07 in all cases. Depending on the unperturbed value of $\sin^2 \theta_{12}$, either positive or negative δ is better compatible with the data for increasing $|\delta|$. All cases apart from $X = S_4$ prefer $\delta < 0$ for the particular choice (3.26). We do not show our results for the other two mixing angles, since in the case of θ_{13} all corrections for $|\delta| \lesssim 0.1$ are small: in the case of S_4 and A_5 $\sin^2 \theta_{13} \lesssim 4 \times 10^{-5}$ holds, compare (3.23), while for patterns M1 and M2 we find $0.041 \lesssim \sin^2 \theta_{13} \lesssim 0.047$ and for M3 and M4 we get $0.0084 \lesssim \sin^2 \theta_{13} \lesssim 0.015$. We clearly see that corrections associated with the non-universality of the neutrino Dirac mass terms are not sufficient in the cases $X = S_4$ and $X = A_5$ to explain $\theta_{13} \sim 0.1 \div 0.2$. For $|\delta| \lesssim 0.1$, the corrections to the atmospheric mixing angle always keep $\sin^2 \theta_{23}$ in its 3σ range [6] for S_4, A_5 and $\Delta(384)$. Since the unperturbed value of $\sin^2 \theta_{23}$ is at the edge of the 3σ range for the patterns M1, $\sin^2 \theta_{23} \approx 0.65$, and M2, $\sin^2 \theta_{23} \approx 0.35$, corrections with $\delta < 0$ for the parametrization (3.26) are welcome because they improve the agreement with the results from global fits. At the same time θ_{12}, see figure 1, remains in its experimentally allowed 3σ range for negative δ with $|\delta| \lesssim 0.04$. Generally speaking, the patterns M3 and M4 are the most promising ones even taking into account corrections coming from the non-universality of the neutrino Dirac mass terms.[9] In the case of the patterns M1 and M2 these corrections help to improve the accordance with the experimental data; however, these patterns are not favoured by the latter. S_4 and A_5 mainly fail to give a good fit to the data because of the too small value of θ_{13}.

The effects of BKT and of non-universal ρ on the lepton mixing, neglected in the above study being sub-leading, become relevant in the $\mathbf{Z}_2$-invariant model, where δ_α vanish. In the latter model we find numerically for S_4 and for A_5 (see caption of figure 2 for details on the chosen parameters)

$$0.32 \lesssim \sin^2 \theta_{12} \lesssim 0.35 \quad \text{and} \quad 0.27 \lesssim \sin^2 \theta_{12} \lesssim 0.29 \,, \tag{3.27}$$

respectively, and

$$0.48 \lesssim \sin^2 \theta_{23} \lesssim 0.52 \,, \tag{3.28}$$

showing that $\sin^2 \theta_{12,23}$ only get corrected by less than 0.03. Corrections to θ_{13} are negligible and thus $\theta_{13} \sim 0.1 \div 0.2$ [6, 7, 8] cannot be achieved in models with S_4 and A_5, by taking into account the BKT or the deviation of the parameters ρ from universality. For $\Delta(96)$, M1 [M2] we get

$$0.63[0.32] \lesssim \sin^2 \theta_{23} \lesssim 0.69[0.36] \,, \tag{3.29}$$

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[9]There is a slight dependence of which of the two patterns performs best on the used global fit analysis: using [6, 8] we find pattern M3 to be the best one, while [7] prefers M4 over M3. This difference originates from the fact that in [7] the best fit value of $\sin^2 \theta_{23}$ is larger than 0.5, whereas it is smaller than 0.5 in [6, 8].
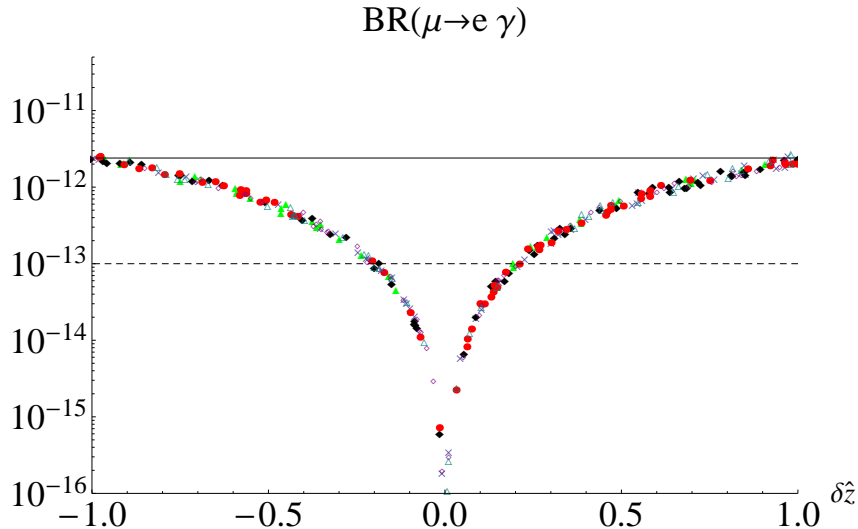
Figure 2: Branching ratio of $\mu \to e\gamma$ as a function of the UV BKT $\delta\hat{z} = 3(\hat{Z}_l)_{e\mu}$ in the $\mathbf{Z}_2$-invariant model. The continuous and dashed lines are the current $(BR(\mu \to e\gamma) < 2.4 \times 10^{-12})$ and the expected future bound $(BR(\mu \to e\gamma) < 10^{-13})$ given by the MEG Collaboration [12], respectively. The parameters c_l, c_ν, h, m_0 and the neutrino mass hierarchy as well as the color coding are chosen as in figure 1. Note that no constraints coming from lepton mixing are taken into account in this data set; however, the constraints coming from gauge coupling deviations are satisfied by each point plotted.

keeping the atmospheric mixing angle compatible with data only at the 3σ level [6], as well as

$$0.34 \lesssim \sin^2\theta_{12} \lesssim 0.37 \quad \text{and} \quad 0.04 \lesssim \sin^2\theta_{13} \lesssim 0.053[0.047] \,. \tag{3.30}$$

For $\Delta(384)$, M3 [M4] we find analogously

$$0.40[0.56] \lesssim \sin^2\theta_{23} \lesssim 0.44[0.59] \,, \tag{3.31}$$

respectively, and

$$0.32 \lesssim \sin^2\theta_{12} \lesssim 0.35 \quad \text{and} \quad 0.01 \lesssim \sin^2\theta_{13} \lesssim 0.012 \,. \tag{3.32}$$

3.2 LFV Processes

In this subsection we examine the bounds coming from LFV processes in all six models proposed (see [21] for early analysis of LFV bounds in warped models). As discussed in detail in [11] and repeated in section 2, these processes are non-negligible only when the leading flavour violating BKT (3.16) at the UV brane are taken into account. The tree-level decay $\mu \to 3e$, $\mu-e$ conversion in nuclei and $\mu \to e\gamma$ depend quadratically on the off-diagonal entry $(e\mu)$ of the matrix (3.17). The relevant combination of $\hat{z}_{\alpha l}$ varies from case to case because Z_l depend on the mixing matrix V. We plot in figure 2 the bounds arising from $\mu \to e\gamma$ as a function of $\delta\hat{z} = 3(\hat{Z}_l)_{e\mu}$ for the $\mathbf{Z}_2$-invariant model,[10] obtained by a numerical computation in which the first KK mode of each

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[10]The factor 3 in the definition naturally arises in the S_4 case, and is left to match the convention used in [11].

tower of states has been kept. $BR(\mu \to e\gamma)$ is essentially the same in all models and shows the expected quadratic dependence on $\delta \hat{z}$. The branching ratio is always below the current experimental limit for $|\delta \hat{z}| \lesssim 1$ and the expected future MEG bound $BR(\mu \to e\gamma) < 10^{-13}$ will require $|\delta \hat{z}| \lesssim 0.2$. Similar results hold for the model with no $\mathbf{Z}_2$ exchange symmetry since the IR masses $m_{IR,\alpha}^{l,\nu}$ do not play an important role. Finally we note that in all models the branching ratios of $\mu \to 3e$, $\mu - e$ conversion in Ti as well as of the radiative τ lepton decays are below their experimental bounds for $|\delta \hat{z}| \lesssim 1$.

3.3 EDMs

Lepton EDMs in the 5D model are completely calculable, because gauge invariance forbids the appearance of uncalculable 5D bulk or boundary operators that reduce, upon KK reduction in 4D, to the EDM operators $(-i/2)\bar{l}_\alpha \sigma^{\mu\nu}\gamma_5 l_\alpha F_{\mu\nu}$. The size of d_α is negligible, due to the relatively few sources of complex parameters. The bulk parameters c_l, c_α and c_ν are real, while the IR and UV mass terms contain in total nine complex phases. Those of the IR mass terms $m_{IR,\alpha}^{l,\nu} = |m_{IR,\alpha}^{l,\nu}|e^{i\theta_\alpha^{l,\nu}}$ can be removed by a simple field redefinition:

$$\xi_{\nu,\alpha} \to e^{-i\theta_\alpha^\nu}\xi_{\nu,\alpha}\,, \qquad \xi_{e,\alpha} \to e^{-i\theta_\alpha^l}\xi_{e,\alpha}\,. \tag{3.33}$$

Note that the terms in the bulk Lagrangian do not change if the transformations (3.33) are applied, because the terms involving the 5D fields $\xi_{l,\alpha}$ and $\xi_{\nu,\alpha}$ are invariant under the flavour symmetry $U(3)$ and those containing $\xi_{e,\alpha}$ under $U(1)^3$. In the field basis of (3.33), all phases are encoded in the UV Majorana mass terms: the phases associated with the UV mass terms $m_{UV,\alpha}$ as well as the phases θ_α^ν. As has been shown in [11] (cf. (4.34)) by considering the KK expansion of the RH neutrinos $\nu_{\alpha R}$, the UV localized Majorana mass term gives effectively rise to only one heavy RH neutrino per generation, with mass of order $10^{12 \div 13}$ GeV, while the remaining orthonormal combinations of KK states are not sensitive to the UV Majorana mass term and have masses setting in at a few TeV. As consequence, the EDMs induced by the UV Majorana mass terms are mediated by these three heavy states and are completely negligible. Notice that the transformations (3.33) do not involve the fields $\xi_{l,\alpha}$ and hence no phases appear in the BKT (3.16). The next-to-leading flavour violating BKT at the UV brane is Z_ν in (2.11), which acquire non-trivial phases after the transformations (3.33). Due to the field localization of $\nu_{\alpha R}$ in the extra dimension, however, the effective BKT are strongly suppressed, $Z_\nu \lesssim 10^{-5}$, if values of c_ν are used which are suitable for reproducing correctly the scale of light neutrino masses, e.g. $c_\nu = -0.365$ like in figures 1 and 2 (this is equivalent to the suppression mechanism of Z_ν explained below (2.11) for a composite ν_R^α). Using the estimate (2.13) with Z_l being replaced by Z_ν, we find that lepton EDMs are well below the current experimental bounds.

4 Conclusions

We have extended the class of 4D HCHM based on the non-abelian flavour group $S_4 \times \mathbf{Z}_3$ introduced in [11] to general non-abelian discrete groups of the form $X \times \mathbf{Z}_N$. This allows to

consider HCHM which predict a promising lepton mixing pattern with non-vanishing $\theta_{13} \sim$ $0.1 \div 0.2$ [9] as favoured by current data. In a 5D scenario with Majorana neutrinos, we have computed in detail the lepton mixing for four particular choices of $X \times \mathbf{Z}_N$. We have shown that flavour symmetry breaking effects at the IR brane affecting neutrinos can be at most 10% in order to not perturb too much the original predictions for the mixing angles. We have argued, like in [11], that a $\mathbf{Z}_2$ exchange symmetry can be imposed on the IR brane avoiding these restrictions. All LFV processes and EDMs are below the experimental bounds.

As discussed in [11], the typical mass scale of the vector-like leptonic fermion resonances in the 5D models is around 2 TeV. The prospects to produce and observe these resonances at the LHC are unfortunately quite limited. A possible signature might be the observation of the decay $\mu \to e\gamma$, considering that its typical branching ratio is within the range of the expected future bound of the MEG experiment.

It would be interesting to extend our considerations to the quark sector and discuss quark mixing in a similar manner.

Acknowledgments

A   Group Theory of S_4, A_5, $\Delta(96)$ and $\Delta(384)$

In this appendix we report some details of the group theory of S_4, A_5, $\Delta(96)$ and $\Delta(384)$. All symmetries can be defined in terms of two generators S and T. We show the relations the latter have to fulfill in order to generate one of the groups, S_4, A_5, $\Delta(96)$ and $\Delta(384)$, and give their explicit realization for an irreducible triplet. The basis chosen for S and T is such that the Lagrangians introduced in (2.5), (3.7) and (3.8) are reproduced. In the following we report the explicit form of S. The generator T can be easily computed using (2.2) and table 2, once the product of S and T giving G_N is known.

In the case of S_4, S and T fulfill the relations [22]

$$S^2 = 1 \ , \quad T^4 = 1 \ , \quad (ST)^3 = 1 \ . \tag{A.1}$$

The explicit form of S can be chosen as

$$S = \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & 1 \\ -2 & 1 & -2 \end{pmatrix} \ . \tag{A.2}$$

The generators of G_e and G_ν are the following

$$G_N = (ST)^2 \ , \quad G_1 = S \ , \quad G_2 = (ST^2)^2 \ . \tag{A.3}$$

The explicit form of T is then

$$T = SG_N^2 = \frac{1}{3} \begin{pmatrix} 1 & -2\,\omega_3 & -2\,\omega_3^2 \\ -2 & -2\,\omega_3 & \omega_3^2 \\ -2 & \omega_3 & -2\,\omega_3^2 \end{pmatrix} . \tag{A.4}$$

We also briefly comment on the relation between the generators S and T used here and the set of generators $\tilde{S}$, $\tilde{T}$ and U used in [11] in order to describe the group S_4. One can check that $\tilde{T}$ is similar to $G_N = (ST)^2$, $\tilde{S}$ to $G_2 = (ST^2)^2$ and U to $G_1G_2 = T^2ST^2$.

The group A_5 is generated through S and T being subject to the conditions [19]

$$S^2 = 1 , \quad T^5 = 1 , \quad (ST)^3 = 1 . \tag{A.5}$$

We choose the realization of S to be

$$S = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\phi & \frac{1}{\phi} \\ \sqrt{2} & \frac{1}{\phi} & -\phi \end{pmatrix} \tag{A.6}$$

with $\phi = (1 + \sqrt{5})/2$ for one of the irreducible triplets. In this case the remnant subgroups G_e and G_ν are generated through

$$G_N = T , \quad G_1 = S , \quad G_2 = T^2ST^3ST^2 . \tag{A.7}$$

The group $\Delta(96)$ is generated through S and T fulfilling the relations [9]

$$S^2 = 1 , \quad T^8 = 1 , \quad (ST)^3 = 1 , \quad (ST^{-1}ST)^3 = 1 . \tag{A.8}$$

In order to realize the mixing pattern called M1, the most convenient choice of basis for S for a faithful irreducible triplet is

$$S = \frac{1}{3} \begin{pmatrix} -1+\sqrt{3} & -1 & -1-\sqrt{3} \\ -1 & -1-\sqrt{3} & -1+\sqrt{3} \\ -1-\sqrt{3} & -1+\sqrt{3} & -1 \end{pmatrix} . \tag{A.9}$$

For the other mixing pattern M2 it is convenient to choose S like in (A.9), however with second and third rows and columns exchanged, respectively. This is clear because the mixing matrices V associated with M1 and M2 are related by the exchange of the second and third rows. Independently of the mixing pattern, the remnant subgroups G_e and G_ν are generated through [9]

$$G_N = ST , \quad G_1 = S , \quad G_2 = (ST^4)^2 . \tag{A.10}$$

Similarly, the group $\Delta(384)$ is generated with S and T fulfilling [9]

$$S^2 = 1 , \quad T^{16} = 1 , \quad (ST)^3 = 1 , \quad (ST^{-1}ST)^3 = 1 . \tag{A.11}$$

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Again, it is convenient to choose two different bases for a faithful irreducible triplet representation in order to generate the mixing patterns M3 and M4, respectively. For a model incorporating the pattern M3 we choose

$$S = \frac{1}{6} \begin{pmatrix} -2+\sqrt{2}+\sqrt{6} & -2(1+\sqrt{2}) & -2+\sqrt{2}-\sqrt{6} \\ -2(1+\sqrt{2}) & -2+\sqrt{2}-\sqrt{6} & 2(-1+\sqrt{2}+\sqrt{3}) \\ -2+\sqrt{2}-\sqrt{6} & 2(-1+\sqrt{2}+\sqrt{3}) & -2(1+\sqrt{2}) \end{pmatrix} . \tag{A.12}$$

For an explicit model leading to the pattern M4, the basis in which S is like in (A.12) with second and third rows and columns exchanged, respectively, is the most appropriate one. Again, this is obvious considering the relation of the mixing patterns M3 and M4. Independently of the mixing pattern, the remnant subgroups G_e and G_ν are generated through [9]

$$G_N = ST , \quad G_1 = S , \quad G_2 = (ST^8)^2 . \tag{A.13}$$

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